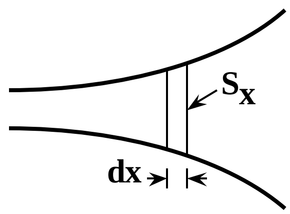
(14.7) Horn Loudspeakers

Let's now consider a horn termination to a pipe.



A horn is an acoustic transformer. The equation of continuity $s = -\frac{\partial \xi}{\partial x}$ for plane waves (when we assumed a constant cross-sectional area) becomes

$$s = -\frac{1}{S_x}\frac{\partial(S_x \xi)}{\partial x}.$$

This will be true as long as the fractional change in S_x is small over distances of a wavelength. The wave equation, then, becomes

$$\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial}{\partial x}\left(\frac{1}{S_x}\frac{\partial(S_x \xi)}{\partial x}\right).$$

We must assume a shape for the horn. Let's assume an exponential shape (exponential horn) where

$$S_x = S_o e^{2bx}$$

where $2b$ is the flare constant and S_o is the initial throat area.

Substituting $S_x = S_o e^{2bx}$ into $\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial}{\partial x}\left(\frac{1}{S_x}\frac{\partial(S_x \xi)}{\partial x}\right)$ yields:

$$\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial}{\partial x}\left(\frac{1}{S_x}\frac{\partial(S_x \xi)}{\partial x}\right) = c^2 \frac{\partial}{\partial x}\left(\frac{1}{S_o e^{2bx}}\frac{\partial(S_o e^{2bx} \xi)}{\partial x}\right)$$

$$= c^2 \frac{\partial}{\partial x}\left(\frac{1}{S_o e^{2bx}}\left(S_o e^{2bx}\frac{\partial \xi}{\partial x} + 2b S_o e^{2bx} \xi\right)\right)$$

$$= c^2 \frac{\partial}{\partial x}\left(\frac{\partial \xi}{\partial x} + 2b\xi\right) = c^2\left(\frac{\partial^2 \xi}{\partial x^2} + 2b\frac{\partial \xi}{\partial x}\right)$$

Therefore, $\frac{\partial^2 \xi}{\partial t^2} = c^2\left(\frac{\partial^2 \xi}{\partial x^2} + 2b\frac{\partial \xi}{\partial x}\right)$

Assume $\tilde{x}(x,t)=\tilde{A}e^{jwt}e^{jgx}$ and substitute into $\frac{\partial^2 x}{\partial t^2}=c^2\left(\frac{\partial^2 x}{\partial x^2}+2b\frac{\partial x}{\partial x}\right)$ yields:

$$-w^2\tilde{x}(x,t)=c^2\left(-g^2\tilde{x}(x,t)+j2bg\tilde{x}(x,t)\right)$$
$$-w^2=c^2\left(-g^2+j2bg\right)$$
$$-\left(\frac{w}{c}\right)^2=-g^2+j2bg$$
$$-k^2=-g^2+j2bg$$
$$g^2-j2bg-k^2=0$$
$$g=-\frac{-j2b}{2}\pm\sqrt{\left(\frac{-j2b}{2}\right)^2-\frac{-k^2}{1}}=jb\pm\sqrt{k^2-b^2}$$
$$\tilde{x}(x,t)=\tilde{A}e^{jwt}e^{jgx}$$
$$jg=-b\pm j\sqrt{k^2-b^2}=-a\pm jk$$

where $a=b$ and $k=\sqrt{k^2-b^2}$

Therefore, $\tilde{x}(x,t)=\tilde{A}e^{jwt}e^{jgx}=\tilde{A}e^{jwt}e^{(-a\pm jk)x}=e^{-ax}\left\{Ae^{j(wt-kx)}+Be^{j(wt+kx)}\right\}$

Phase velocity:

$$c_p=\frac{w}{k}$$

The phase velocity (or phase speed) is the speed of the pressure maximum through the horn and is a function of frequency (dispersion).

$$c_p=\frac{w}{k}=\frac{w}{\sqrt{k^2-b^2}}=\frac{w}{k\sqrt{1-\frac{b^2}{k^2}}}=\frac{c}{\sqrt{1-\left(\frac{b}{k}\right)^2}}$$

As f decreases, k increases, and from $jg=-b\pm j\sqrt{k^2-b^2}=-a\pm jk$, when $k^2=b^2$ or $\left(\frac{b}{k}\right)^2=1$,

there is a cut-off frequency, f_c. Observe:

From $c_p=\frac{c}{\sqrt{1-\left(\frac{b}{k}\right)^2}}$, as f decreases to $1-\left(\frac{b}{k}\right)^2=0$, this leads to $c_p\to\infty$.

This indicates that all parts of the medium in the horn move in phase, that is, the motion at the mouth is in phase with the motion at the throat.

Group velocity:

$$c_g=\frac{dw}{dk}$$

The group velocity (or group speed) is the speed of the energy transport through the horn. From

$$k = \sqrt{k^2 - b^2}\ ,$$

$$k^2 = k^2 - b^2 = \frac{w^2}{c^2} - b^2$$

$$\frac{w^2}{c^2} = k^2 + b^2$$

$$w^2 = c^2\left(k^2 + b^2\right)$$

$$w = \pm c\sqrt{k^2 + b^2}$$

$$c_g = \frac{dw}{dk} = \frac{d}{dk}\left\{\pm c\sqrt{k^2 + b^2}\right\} = \pm c\frac{1}{2\sqrt{k^2 + b^2}}\frac{d}{dk}\left(k^2 + b^2\right)$$

$$= \frac{\pm ck}{\sqrt{k^2 + b^2}} = \frac{\pm c\sqrt{k^2 - b^2}}{\sqrt{\left(k^2 - b^2\right) + b^2}} = \frac{\pm ck\sqrt{1 - \left(\frac{b}{k}\right)^2}}{k}$$

$$= \pm c\sqrt{1 - \left(\frac{b}{k}\right)^2} = c\sqrt{1 - \left(\frac{b}{k}\right)^2}$$

As f decreases, k increases, and from $jg = -b \pm j\sqrt{k^2 - b^2} = -a \pm jk$, when $k^2 = b^2$ or $\left(\frac{b}{k}\right)^2 = 1$, there is a cut-off frequency, f_c. Observe:

From $c_g = c\sqrt{1 - \left(\frac{b}{k}\right)^2}$ for which $k = b$, $c_g \to 0$.

Waves below a cutoff frequency do not propagate, i.e., $k = b$ defines cutoff

$$f_c = \frac{b\,c}{2p}$$

Acoustic impedance in horn

$$\tilde{Z}(x) = \frac{r_0 c}{S(x)}\frac{1}{k}\frac{\left(k + jb\right)\tilde{A}e^{-jkx} - \left(k - jb\right)\tilde{B}e^{jkx}}{\tilde{A}e^{-jkx} + \tilde{B}e^{jkx}}$$

Nonzero $\tilde{B}$ (refl. wave) because horn is finite length, however, the reflection is small for $ka > 3$, (typical value to assume near perfect matching) where a is the radius of the mouth.

Setting $\tilde{B} = 0$

$$\tilde{Z}(0) = \frac{r_0 c}{S_o k}(k + jb) = \frac{r_0 c}{S_o}\left\{\sqrt{1-\left(\frac{b}{k}\right)^2} + j\frac{b}{k}\right\}$$

******************************* Example 14.1 *******************************

radius of throat	= 0.02 m	$2b$	= 3.74
radius of mouth	= 0.4 m	f_c	≅ 100 Hz
length is	= 1.6 m		

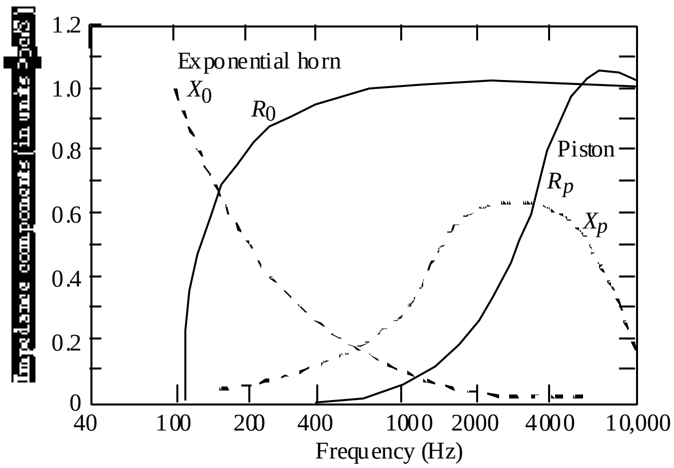


Figure 14.7.2 The acoustic resistance and reactance acting at the throat of an infinite exponential horn (R_0 and X_0) and on a piston mounted in an infinite baffle (R_p and X_p).

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Attenuation coefficient:

$$a = b$$

Further, observe from:

$$\tilde{x} = e^{-ax}\left\{Ae^{j(wt-kx)} + Be^{j(wt+kx)}\right\} = e^{-bx}\left\{Ae^{j\left(wt-\sqrt{k^2-b^2}x\right)} + Be^{j\left(wt+\sqrt{k^2-b^2}x\right)}\right\}$$

There are 3 separate cases: $k > b$, $k = b$, and $k < b$

1. k > β

Propagated wave attenuates with distance exponentially

$$c_p = \frac{c}{\sqrt{1-\left(\frac{b}{k}\right)^2}}$$

$$c_g = c\sqrt{1-\left(\frac{b}{k}\right)^2}$$

2. $k = b$ (cut-off frequency)

$c_p \to \infty$

$c_g \to 0$

3. $k < b$

$$\tilde{x} = e^{-bx}\left\{Ae^{j\left(wt-\sqrt{k^2-b^2}x\right)} + Be^{j\left(wt+\sqrt{k^2-b^2}x\right)}\right\} = e^{-bx}\left\{Ae^{+\sqrt{b^2-k^2}x} + Be^{-\sqrt{b^2-k^2}x}\right\}e^{jwt}$$

No propagation

Transducers

(l4.3a) Transducers

<u>Electrostatic Transducer</u> (reciprocal):

Is modeled as a pair of capacitor plates, one plate is held stationary while the other plate (the diaphragm) moves in response to mechanical or electrical excitation. The transducer is connected to an external circuit with V_0 a constant polarization voltage, C_B a capacitor blocking direct current from the electrical terminals, and R_B a resistor isolating the polarization source from the alternating current.

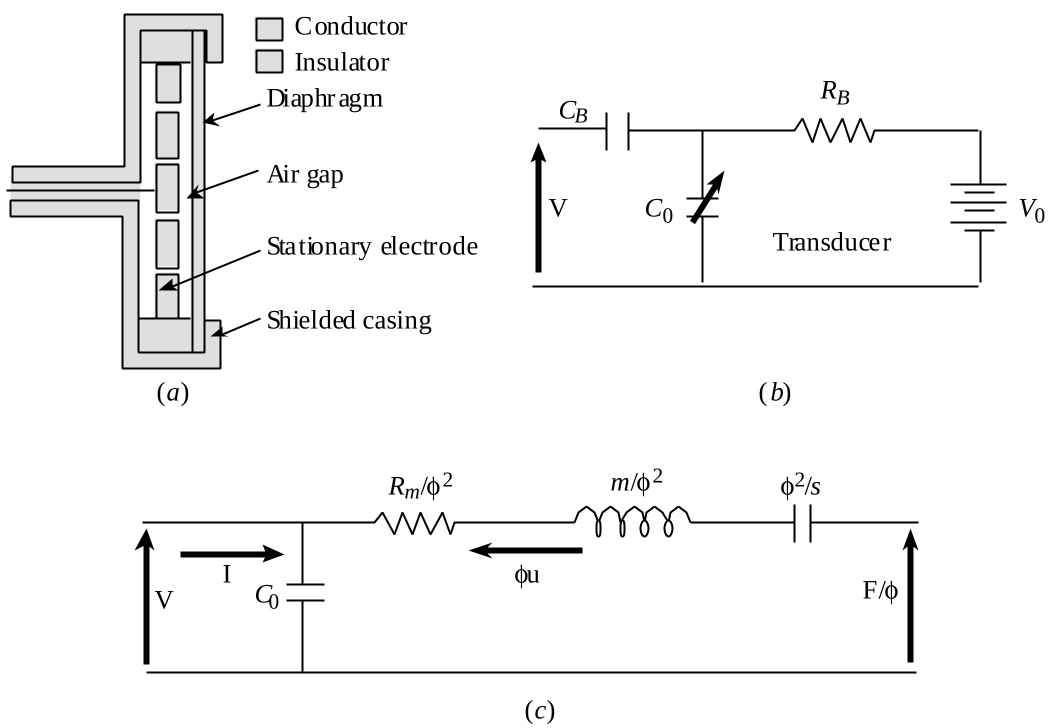


Figure 14.3.1 The electrostatic transducer as a representative reciprocal transducer. (*a*) Schematic of the construction of the transducer. (*b*) External circuit including the capacitance C_0 of the transducer. (*c*) Equivalent circuit.

If we set up our circuit so that $C_B >> C_0$ and $R_B >> \frac{1}{wC_B}$ we can neglect C_B and R_B for an ac signal.

If an ac voltage is applied across the plates, the diaphragm moves in response to the changing charge. Likewise, if the diaphragm encounters an incident pressure field, the motion of the diaphragm will generate an electrical signal. The capacitance of the transducer with the plates at rest is given by

$$C_0 = \frac{eS}{x_0}$$

e – permittivity or dielectric constant
s – surface area
x_0 – equilibrium spacing between plates

$q_0 = C_0 V_0$
$\tilde{V}$ – sinusoidal voltage

So, when we superimpose a sinusoidal voltage on our polarization voltage we have

$$\tilde{V} + V_0 = (\tilde{q} + q_0) \Big/ \left(\frac{eS}{\tilde{x} + x_0} \right)$$

For $\tilde{q} \ll q_0$ and $\tilde{x} \ll x_0$, $\tilde{V}, \tilde{q}$ and $\tilde{x}$ are sinusoidal and we can define a current and velocity as

$$\tilde{u} = jw\tilde{x} \text{ and } \tilde{I} = jw\tilde{q}$$

This gives for our sinusoidal voltage

$$\tilde{V} = \frac{1}{jwC_0}\tilde{I} + \frac{V_0}{jwx_0}\tilde{u}$$

So the voltage depends on the applied current and the motion of the diaphragm. Reciprocity tells us

$$\tilde{F} = \frac{V_0}{jwx_0}\tilde{I} + Z_{mo}\tilde{u}$$

where Z_{mo} is the open-circuit mechanical impedance. Comparing the voltage and force

$$T = T_{em} = T_{me} = \frac{V_0}{j\omega x_0}$$

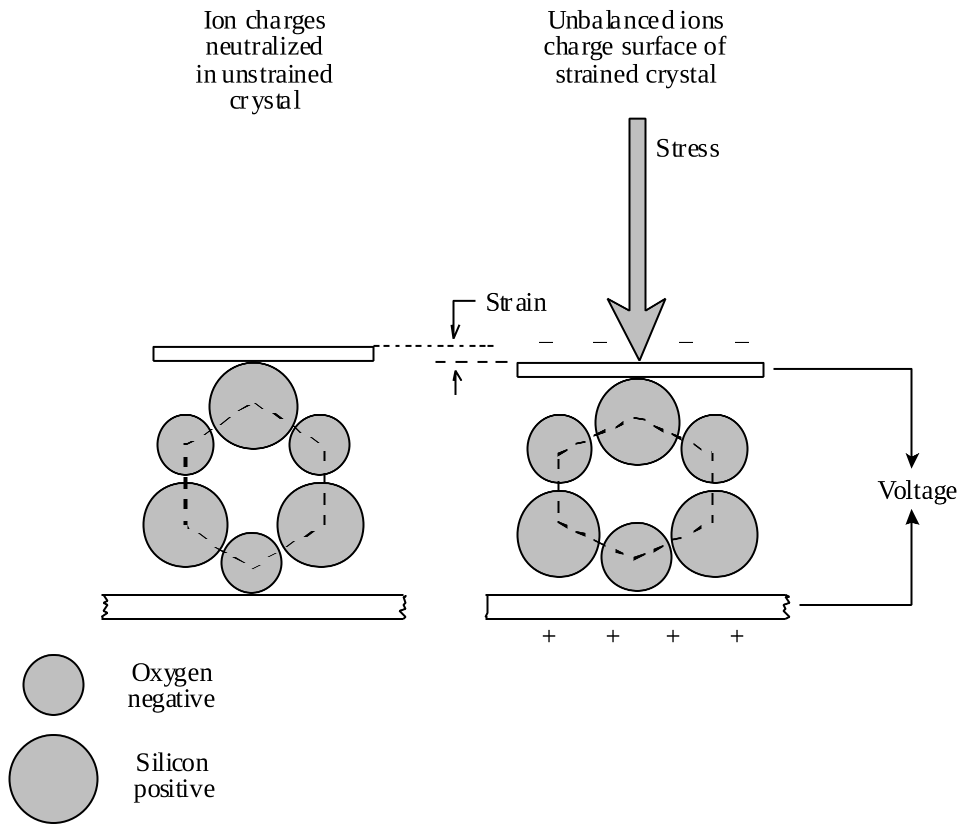
where T is the transduction coefficient.
What happens if you do not apply a sinusoidal voltage?

Piezoelectric (reciprocal transducer)

Certain crystals and ceramics

Voltage causes molecular rearrangement $\Rightarrow$ face of crystal moves
Stress (pressure) on face causes rearrangement $\Rightarrow$ charge (voltage) developed

For quartz



Moving Coil Transducer (antireciprocal)

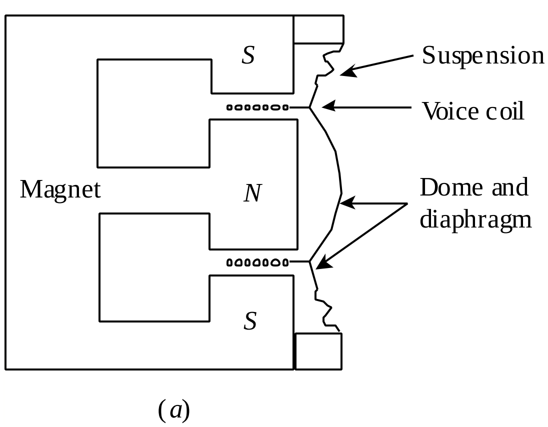
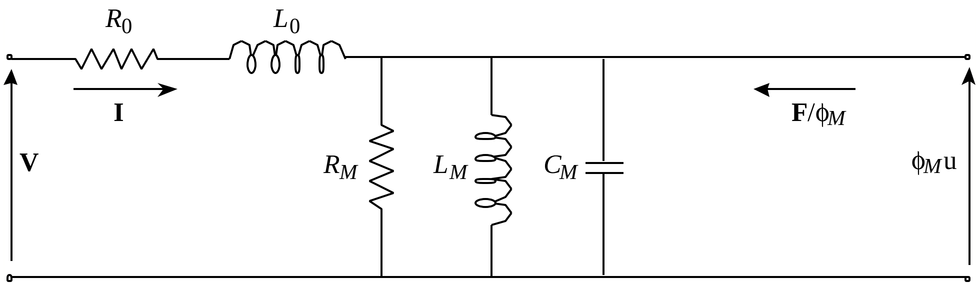


Figure 14.3.2 The moving-coil transducer as a representative antireciprocal transducer. (*a*) Schematic of the construction of the transducer. (*b*) Equivalent circuit.



(*b*)

a.c. voltage generates current in coil mounted in magnetic field which causes coil to move. Similarly, movement of coil will generate a voltage (emf) in coil.

In this case it is anti-reciprocity and

$$T = T_{em} = -T_{me}$$